Natural Sciences 102 -- Spring 2005
Exam #2, May 24, 2005

Name (PLEASE print legibly): Ima Key

1 (10)	2 (20)	3 (10)	4 (10)	5 (10)	6 (10)	7 (10)	8(10)	Total (80)

General instructions:

- For essay and descriptive questions, please be complete, but concise. Answers should be limited to the space provided under the question.
- Please manage your time. First answer the questions you know best.
- **Each problem is worth 10 points.**
- Some formulae you may find useful:

apparent magnitudes of objects 1 & 2 (I = intensity): $m_1 - m_2 = -2.5\log\left(\frac{I_1}{I_2}\right)$

apparent magnitude of the sun: -26.8

$1 \text{ pc} = 200{,}000 \text{ AU}$ speed of light: 300,000 km s^{-1}

Hubble's law: $\text{v} = H_0 d$ speed of sound: 700 miles per hour

speed of smell: 200 inches per minute

shift in wavelength due to motion (c = velocity of the wave): $\frac{\lambda}{\lambda_0} = 1 \pm \frac{\text{v}}{c}$

Inverse square law: intensity $= \left(\frac{\text{luminosity}}{4\pi \text{R}^2}\right)$

Luminosity of the sun: 10^{26} Watts Seconds in a year 3×10^{7}

Human years in a dog year 7

$\left(\frac{\text{distance}}{\text{pc}}\right) = \left(\frac{\text{seconds}}{\text{parallax}}\right)$ $1 \text{ Mpc} = 3\times10^{19} \text{ km}$

I. The Cosmological Distance Scale:

Things you can measure about stars and galaxies are

1) they appear to move
2) they have different apparent brightness
3) they appear to have different colors
4) they change in brightness
5) they are redshifted or blueshifted

For each, give an example of how the properties are used in the construction of the distance ladder.

(1) They appear to move: annual stellar parallax

(2) Stars appear to have different brightness:
An object's brightness (or intensity *I*) depends on both its intrinsic luminosity *L* and its distance *D* from us, through the inverse square law:

$$I = \frac{L}{4\pi D^2}.$$

Knowing the brightness of an object whose distance is known from another method (such as parallax) allows one to compute the intrinsic luminosity.

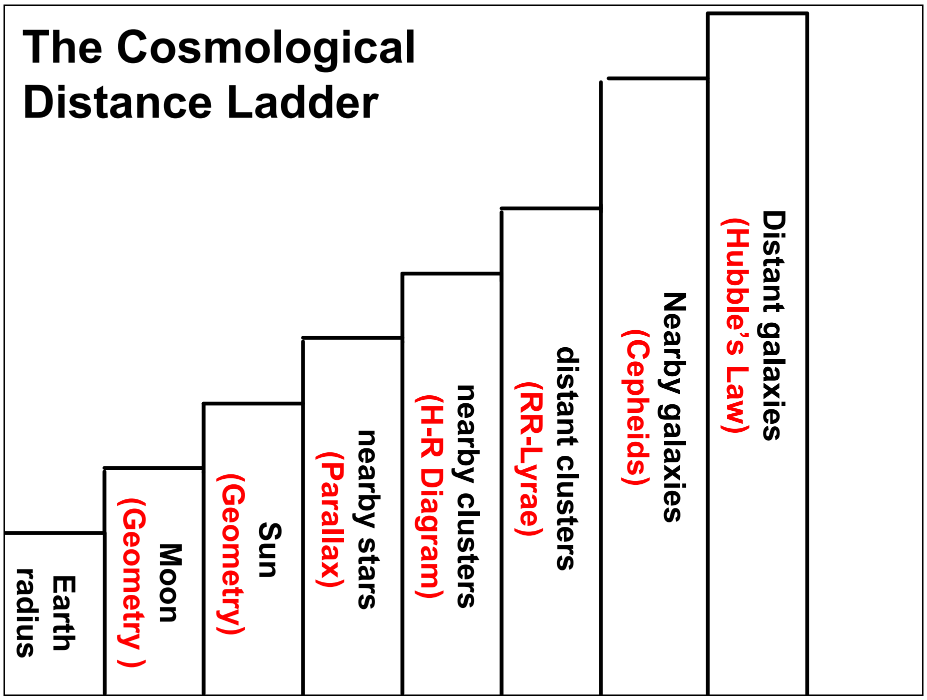
(3) Stars have different colors:
Knowing a star's color (its spectral type) in addition to its brightness allows one to construct the Hertzsprung-Russell (HR) diagram, showing color vs. brightness (as shown in Problem III). For stars in a single cluster (i.e., at approximately the same, known distance), the stars fall in a particular and recognizable pattern. Plotting the stars from a different cluster (thus at a different distance), the pattern will be displaced in brightness from the first. The difference in intensity allows one to use the inverse square law to determine the distance).
(Also technically acceptable was Hubble's Law. Due to the expansion of the universe, the spectrum (or "color") of a galaxy gets redshifted, making it appear that it is moving with some velocity v. The distance to the galaxy is then given from Hubble's Law: $D = H_0^{-1}\mathrm{v}$.)

(4) Stars may have changes in brightness:
RR Lyrae and Cepheid stars are easily identified by their pattern of variability (change in brightness). RR Lyrae stars all have approximately the same intrinsic luminosity, while Cepheids have a luminosity which is related to the measured period of variability. Having identified one of these stars in a cluster whose distance is known (by some other method), the exact value of the luminosity can then be used to use these as standard candles.

(5) They are redshifted or blueshifted:
Hubble's law relates distance and redshift.

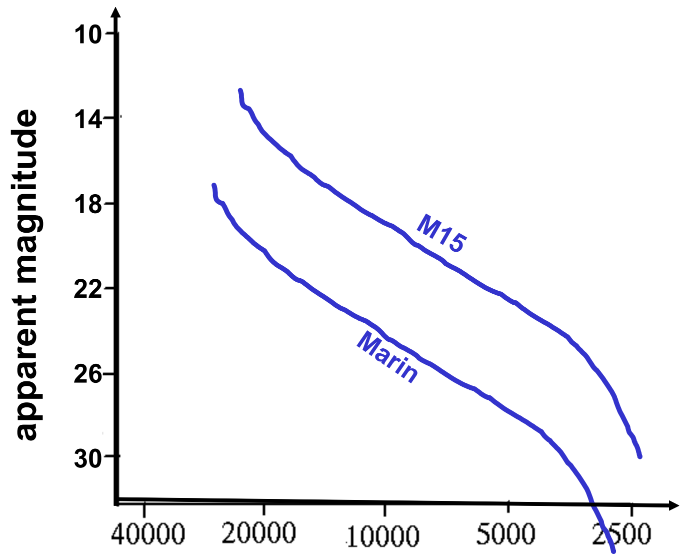
II. The Cosmological Distance Scale:

a) For each step in the distance ladder, fill in (in the parentheses) the method used to determine the distance (for the sun & moon, the original method). Choose from Cepheid Stars, Faith-Based Determination, Geometry, H-R Diagram, Hubris, Hubble's Law, Parallax, RR-Lyrae Stars, Radio Dating, Internet Dating.



III. The H-R Diagram as a distance indicator:

a) Describe in words (use equations if you wish) how the H-R diagram can be used as a distance indicator.
b) The distance to the cluster M15 is known to be 1000 pc. Another cluster is discovered by Felipe, named the Marin cluster. Use the information from the H-R diagrams below to calculate the distance to Marin.



3a) Many stars must be measured to establish a good HR diagram. Measuring only one star will not do. We assume that properties of globular clusters are similar and thus the overall trend of the HR diagram should be the same for all globular clusters. We compare HR diagram of Marin with an HR diagram of a cluster whose distance is known, say M15. Plot intensity (apparent magnitude) in the y-axis. Marin and M15 have the same **intrinsic** L luminosity, overall and at each particular temperature, but due to their distance d, their apparent luminosity does not appear to be the same.

$I_1 = \dfrac{L}{4\pi d_1^2}$, $I_2 = \dfrac{L}{4\pi d_2^2}$ since L_1=L_2, we can calculate the distance:

$I_1 d_1^2 = I_2 d_2^2 \rightarrow d_1 = \sqrt{\dfrac{I_2}{I_1}}\, d_2$. The intensities can be determined by the equation $m_1 - m_2 = -2.5\log\left(I_1/I_2\right)$. These two equations can be combined to give the complete equation relating distance to apparent magnitude at a given temperature, $m_1 - m_2 = -5\log\left(d_2/d_1\right)$.

3b) The magnitudes can be chosen at any temperature. The following is an example solution. At about 20,000 Kelvin, M15 has an apparent magnitude of about 15. Marin has one at about 20. The distance to M15 is known to be 1000 pc. Therefore the distance to Marin is given by $20 - 15 = -5\log\left(1000\text{ pc}/d_{\text{Marin}}\right)$ where the 1 represents Marin, and the 2 represents M15. So d_{Marin} is about 10,000 pc.

IV. Parallax:

a) What is annual stellar parallax? What moves, and what appears to move?
b) Draw a diagram indicating the parallax angle, the distance known by other means and the distance determined by parallax.
c) The annual stellar parallax of the star Donaghy is measured to be 0.01 seconds of arc. What is its distance?

a) Annual Stellar Parallax is the apparent movement of a nearby star with respect to the more distant background stars. Maximum apparent displacement comes at roughly 6 month intervals, when the Earth is on opposite sides of the sun. In a sense, this parallax is an optical illusion--the stars appear to move, but it is the Earth which is really moving.

b) This should be some sort of diagram indicating that parallax is best measured from the Earth six months apart, in order to give the longest baseline (1AU) for the measurement. This diagram should look very similar to the diagram shown many times in class, e.g., on page 12 of lecture 8 (April 26).

c) You may certainly use the equation $\tan\alpha = 1\ \mathrm{AU}/D$, but this is more difficult than it needs to be. If you do use this equation, remember there is no need to convert your angle from degrees to radians. Provided your calculator knows whether it is getting degrees or radians, either may be entered in your calculator.

For small angles (and given that the closest star to the sun is at about 1 pc and, therefore, has a parallax angle of 1 arc second, all stellar parallax angles are small), you may use the simpler formula $\alpha = 1\ \mathrm{AU}/D$, which implies $D = 1\ \mathrm{AU}/\alpha$. Remember that, in this formula, distances will be in parsecs if the angle is in arc seconds. This problem just becomes:

$$D = 1/.01 = 100\ \text{pc}.$$

V. Doppler Shift:

a) Define the Doppler shift and explain how it can be used to determine velocity.

b) Andrew Hill is driving his new pickup truck around Hyde Park to impress his girlfriend. He is pulled over for running a red light. He claims the light looked green to him because of the Doppler effect. The wavelength of red light is 6000 Angstroms and the wavelength of green light is 4500 Angstroms.

1. In order for a red light to appear green, would Andrew have to be traveling toward the light or away from the light?
2. Estimate Andrew's speed for a red light to appear green?

a) The Dopper shift is the shifting of emitted wavelengths (frequency) due to either the source or the observer moving. This holds for all forms of wave, including light and sound.

The change of wavelength is proportional to the velocity of the source or the observer; the faster the velocity, the greater the shift in wavelength.

b) 1. Andrew is moving towards the light.

2. Since Andrew is moving towards the object, the equation we need to use is

$$\frac{\lambda - \lambda_0}{\lambda_0} = -\frac{\mathrm{v}}{c}$$

- Emitted wavelength (red): $\lambda_0 = 6000 \text{ Angstroms}$.
- Observed wavelength (green): $\lambda_0 = 4500 \text{ Angstroms}$.

The speed Andrew was traveling is

$$\frac{4500 - 6000}{6000} = -\frac{\mathrm{v}}{c}$$

$$\frac{-1500}{6000} = -\frac{\mathrm{v}}{c}$$

$$0.25 = -\frac{\mathrm{v}}{300{,}000 \text{ km s}^{-1}}$$

VI. Cosmological Principle:

a) Please state the cosmological principle.
b) What are the implications of the cosmological principle for the location of the center and the edge of the universe?
c) Reconcile the observational fact that most galaxies are receding from us with the cosmological principle. Why doesn't that fact make our location special?
d) Is every galaxy redshifted?

a)
The cosmological principle can be stated in two ways:

i. There is no preferred point in the Universe and there is no preferred set of points in the Universe
ii. The Universe in large scales is homogeneous and isotropic

b)
The cosmological principle implies that there is neither a center in the universe because that would be neither a special point nor a boundary because that would be a special set of points.

c)
The fact that we see almost all galaxies receding from ours, gives us the impression that we are the center of the universe. However, if that were true it would violate the cosmological principle. This observation is reconciled with the cosmological principle because no matter where in the universe one is, one would see all galaxies moving away from him. Thus, the universe is a democratic place, since every observer in it sees the same picture.

VII. Magnitudes:

a) Why is the magnitude scale based on a lograrithmic scale?
b) Which appears brighter, a star of 12th apparent magnitude or a star of −12th apparent magnitude?
c) The star Cheney-V has an apparent magnitude of m = −1. Another star, Daley-7, is the same distance from us as Cheney-V, but has a luminosity of one hundred times the luminosity of Cheney-V. What is the apparent magnitude of Daley-7?

a) It is based on the fact that our senses respond logarithmically to stimuli.

b) The −12th magnitude star is brighter, for the smaller the magnitude the brighter the source.

c) The formula for magnitudes is

$$m_D - m_C = -2.5\log\left(\frac{I_D}{I_C}\right) = -2.5\log\left(\frac{L_D/4\pi R_D^2}{L_C/4\pi R_C^2}\right).$$

Now we know $m_C = -1$, $L_D = 100L_C$

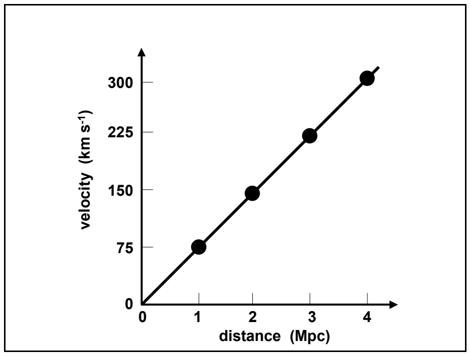
$$m_D - (-1) = -2.5\log\left(\frac{L_D}{L_C}\right)$$

$$m_D + 1 = -2.5\log(100) = -2.5\times 2 = -5$$

$$m_D = -6$$

VIII. Hubble's Law

1) The equation of Hubble's law is on the cover page. Tell me in words what the symbols mean.
2) From the following velocity-distance diagram, calculate the Hubble constant in units of km s^{-1} Mpc^{-1}.



3) Please explain how the above diagram can be used as a basis for an estimate of the age of the universe.
4) Using the information in the diagram above, what is expansion age of the universe?

1) $v = H_0 d$: v = velocity of recession, H_0 = Hubble's constant, d = distance to the galaxv.

2) Can use any of the points or just calculate the slope of the line. For a galaxy 1 Mpc distance the recessional velocity is 75 km s^{-1} Mpc-1, so H_0= 75 km s^{-1} Mpc^{-1}.

3) One can recall distance = velocity $\times$ time. Hubble's law is $v = H_0 d$, which can be rewritten as $d = v / H_0$. One may then identify $time = 1 / H_0$.

4)
$$\text{age} = \frac{1}{H_0} = \frac{1}{75 \text{ km s}^{-1} \text{ Mpc}^{-1}} = \frac{1}{75} \frac{\text{Mpc}}{\text{km}} \frac{1}{\text{s}}$$
$$= \frac{1}{75} \frac{\text{Mpc}}{\text{km}} \frac{3\times10^{19} \text{km s}}{1 \text{ Mpc}} \frac{1 \text{yr}}{3\times10^{7} \text{s}}$$
$$= 13.3\times10^{9} \text{ yr}$$